zoid $PQHM$ is isosceles. $\therefore PM=QH$, but $PM=LB$. $\therefore LB=QH$ and is parallel to it since $\angle PLB=\angle PQH=\angle PEF$. $\therefore EF$ is parallel to QH.

Now E is the midpoint of PQ, hence O is the midpoint of PH.

Also solved by the Proposer.

304. Proposed by G. W. GREENWOOD, M. A., Dunbar, Pa.

Find the tangent at the points $(a, 0)$ and $(0, a)$ to the locus $x^3+y^3=a^3$, and show that these points are points of inflexion.

I. Solution by A. H. HOLMES, Brunswick, Maine.

$x^3+y^3=a^3$. $\therefore \dfrac{dy}{dx}=-\dfrac{x^2}{(a^3-x^3)^{2/3}}$, which is 0 for $x=0$, and ∞ for $x=a$.

$\dfrac{d^2y}{dx^2}=-\dfrac{2x(a^3-x^3)^{1/3}-2x^4}{a^3-x^3}$, which is 0 for $x=0$, and ∞ for $x=a$. Take $x>a$,

and $\dfrac{d^2y}{dx^2}$ is seen to be minus. Take $x<a$ (a little) and $\dfrac{d^2y}{dx^2}$ is plus.

$\therefore (a, 0)$ and $(0, a)$ are points of inflexion.

II. Solution by BENJ. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

We have for the slope of the curve at any point, $\dfrac{dy}{dx}=-\dfrac{x^2}{y^2}$. $\therefore y-y_1=-\dfrac{x_1^2}{y_1^2}(x-x_1)$ is the equation of the tangent at any arbitrary point (x_1, y_1) of the curve. For $(0, a)$, the equation of the tangent is $y-a=0$. For $(a, 0)$, the equation of the tangent is $x-a=0$. From the equation of the tangent we have $y=y_1-\dfrac{x_1^2}{y_1^2}(x-x_1)$. In this equation, find y, for $x=x_1-h$ and $x=x_1+h$; also find the corresponding values of y from the equation of the curve, $y=\sqrt[3]{(a^3-x^3)}$. If the differences of these corresponding values of y change signs, the point is a point of inflection; if they do not, the point is an ordinary point of tangency. From the equation of the tangent, the values of y for the point $(0, a)$ are $y'=a$, $y''=a$, and from the curve $'y=\sqrt[3]{(a^3+h^3)}$, $''y=\sqrt[3]{(a^3-h^3)}$; $'y-y'=\sqrt[3]{(a^3+h^3)}-a>0$, $'y-y'=\sqrt[3]{(a^3-h^3)}-a<0$. Hence, $(0, a)$ is a point of inflection.

Similarly for the point $(a, 0)$, $y'=\infty$, $y''=-\infty$.
$'y=\sqrt[3]{(3a^2h-3ah^2+h^3)}$, $''y=\sqrt[3]{(-3a^2h-3ah^2-h^3)}$,
$'y-y'=\sqrt[3]{(3a^2h-3ah^2+h^3)}-\infty<0$, $''y-y''=\sqrt[3]{(-3a^2h-3ah^2-h^3)}+\infty>0$.

Hence, the point $(a, 0)$ is a point of inflection.

Also solved by G. B. M. Zerr, J. Scheffer, and the Proposer.